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Take $AB=b$; with O the center of AB as center and a radius equal to m, describe the circle HCL. Construct the angles ABH and HBC equal to the angles KEF and FEG, respectively.

Through $ABCH$ describe a circle. Join AH, OH, OC, AC. Then ABC or ABH is the required triangle, since $AB=b$, $OH=OC=m$, $\angle HCB=\angle HAC=D$.

CALCULUS.

102. Proposed by G. B. M.ZERR, A.M., Ph.D., Professor of Chemistry and Physics.The Temple College,Philadelphia. Pa.

A right cone has its vertex at the focus of a paraboloid of revolution, the axis of the cone perpendicular to the axis of the paraboloid. Find the volume common to both.

Solution by the PROPOSER.

Let $x^2+y^2=c^2z^2$, be the equation to the cone. $y^2+z^2=4a(a+x)$, be the equation to the paraboloid.

The limits of z are $z=\dfrac{\sqrt{(x^2+y^2)}}{c}$ to $z=\sqrt{(4a^2+4ax-y^2)}$;

of y, 0 and $\sqrt{\dfrac{4a^2c^2+4ac^2x-x^2}{1+c^2}}=y_1$;

of x, $2ac[c-\sqrt{(1+c^2)}]=x_2$ to $2ac[c+\sqrt{(1+c^2)}]=x_1$.

$$\therefore\ V=2\int_{x_2}^{x_1}\left(\sqrt{(4a^2+4ax-y^2)}-\frac{\sqrt{(x^2+y^2)}}{c}\right)dxdy$$

$$=\int_{x_2}^{x_1}\Bigg[\Bigg[y\sqrt{[4a^2+4ax-y^2]}+4a(a+x)\sin^{-1}\left(\frac{y}{2\sqrt{[a(a+x)]}}\right)-\frac{y\sqrt{[x^2+y^2]}}{c}$$

$$-\frac{x^2}{c}\log[y+\sqrt{(x^2+y^2)}]\Bigg]_0^{y_1}\Bigg]dx=4a\int_{x_2}^{x_1}(a+x)\sin^{-1}\left(\frac{\sqrt{[4ac^2(a+x)-x^2]}}{2\sqrt{[a(1+c^2)(a+x)]}}\right)dx$$

$$-\frac{1}{c}\int_{x_2}^{x_1}x^2\log\left(\frac{\sqrt{[4ac^2(a+x)-x^2]}+c(2a+x)}{x\sqrt{(1+c^2)}}\right)dx.$$

$$\therefore\ V=\Bigg[2a(a+x)^2\sin^{-1}\left(\frac{\sqrt{[4ac^2(a+x)-x^2]}}{2\sqrt{[a(1+c^2)(a+x)]}}\right)$$

$$-\frac{x^3}{3c}\log\left(\frac{\sqrt{[4ac^2(a+x)-x^2]}+c(2a+x)}{x\sqrt{(1+c^2)}}\right)\Bigg]_{x_2}^{x_1}+a\int_{x_2}^{x_1}\frac{x(a+x)dx}{\sqrt{[4ac^2(a+x)-x^2]}}$$

$$+\tfrac{1}{3}\int_{x_2}^{x_1}\frac{x^3dx}{\sqrt{[4ac^2(a+x)-x^2]}}-\frac{1}{3c}\int_{x_2}^{x_1}x^2dx$$

$$-\frac{1+c^2}{3e}\int_{x_2}^{x_1}\frac{x^4\,dx}{\sqrt{[4ac^2(a+x)-x^2]}\{c(2a+x)+\sqrt{[4ac^2(a+x)-x^2]}\}}.$$

Let $2ac^2-x=2ac\sqrt{(1+c^2)}\cos\theta$.

$$\therefore\ V=2a^3c\int_0^\pi[c-\sqrt{(1+c^2)}\cos\theta][1+2c^2-2c\sqrt{(1+c^2)}\cos\theta]d\theta$$

$$+\tfrac{8}{3}a^3c^2\int_0^\pi[c-\sqrt{(1+c^2)}\cos\theta]^3d\theta-\tfrac{8}{3}a^3c^2\sqrt{(1+c^2)}\int_0^\pi[c-\sqrt{(1+c^2)}\cos\theta]^2\sin\theta d\theta$$

$$-\tfrac{8}{3}a^3c^2\sqrt{(1+c^2)}\int_0^\pi\frac{[c-\sqrt{(1+c^2)}\cos\theta]^4d\theta}{\sin\theta-c\cos\theta+\sqrt{(1+c^2)}}.$$

$$\therefore\ V=\frac{2\pi a^3c^2}{3}(6+15c^2+10c^4)-\tfrac{16}{9}a^3c^2\sqrt{(1+c^2)}(1+4c^2)$$

$$-\tfrac{8}{3}a^3c^2\sqrt{(1+c^2)}\int_0^\pi\frac{[c-\sqrt{(1+c^2)}\cos\theta]^4d\theta}{\sin\theta-c\cos\theta+\sqrt{(1+c^2)}}.$$

Let $c=\cot\beta$.

$$\therefore\int_0^\pi\frac{[c-\sqrt{(1+c^2)}\cos\theta]^4d\theta}{\sin\theta-c\cos\theta+\sqrt{(1+c^2)}}=\frac{1}{\sin^3\beta}\int_0^\pi\frac{(\cos\beta-\cos\theta)^4d\theta}{1-\cos(\theta+\beta)}$$

$$=\frac{8}{\sin^3\beta}\int_0^\pi\sin^2\tfrac{1}{2}(\theta+\beta)\sin^4\tfrac{1}{2}(\theta-\beta)d\theta=D.$$

Let $\theta-\beta=2\varphi$. $\therefore\ \theta+\beta=2\varphi+5\beta$.

$$\therefore\ D=\frac{16}{\sin^3\beta}\int_{-\frac{1}{2}\beta}^{\frac{1}{2}(\pi-\beta)}\sin^2(\varphi+\beta)\sin^4\varphi d\varphi$$

$$=\frac{16}{\sin^3\beta}\int_{-\frac{1}{2}\beta}^{\frac{1}{2}(\pi-\beta)}(\sin^6\varphi\cos^2\beta+2\sin\beta\cos\beta\sin^5\varphi\cos\varphi+\sin^4\varphi\cos^2\varphi\sin^2\beta)d\varphi$$

$$=\tfrac{1}{2}\pi\operatorname{cosec}\beta(1+5\cot^2\beta)-\tfrac{2}{3}(1+4\cot^2\beta)=\tfrac{1}{2}\pi\sqrt{(1+c^2)}(1+5c^2)-\tfrac{2}{3}(1+4c^2).$$

$$\therefore\ V=\tfrac{2}{3}\pi a^3c^2(4+3c^2).$$

103. Proposed by HON. JOSIAH H. DRUMMOND, LL. D., Portland, Me.

A park, in the shape of an ellipse whose diameters are 100 and 50 rods, respectively, is surrounded by a wall: one end of a rope, whose length is the circumference of the ellipse, is fastened (outside of the wall) at one end of the longer diameter and the other end at the other end of the same diameter. Over how much surface will a horse graze, which is fastened to a ring moving freely on the rope ?

Remarks by B. F. FINKEL, A. M., M. Sc., Professor of Mathematics and Physics, Drury College, Springfield, Mo.

No correct solution of this problem has yet been received. The problem has been misinterpreted by several of our contributors, who have mistaken it as